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Effect of Atmospheric Processes on Launch Decisions

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The different phases of space-vehicle mission operations and system analyses are influenced by several dynamic atmospheric parameters, such as thunderstorm, precipitation, cloud ceiling, etc., which are interpreted as atmospheric constraints for the launch operations. An atmospheric parameter is a random variable which attains either a permissible (GO) or a not permissible (NOGO) outcome, and in repeated observations these outcomes are not usually independent. The purpose of this paper is to construct analytical models supported by the theory of Markov chains and the theory of runs. Probabilistic models incorporating dependence structure of the Markovian type are analyzed. The underlying theory could be used to predict a GO–NOGO decision in the different phases of a mission. Finally, we present numerical examples of how the developed methods can be used to predict different configurations of GO and/or NOGO outcomes pertaining to thunderstorm activities using 33 years of atmospheric data for Kennedy Space Center.

Nomenclature

$\binom{n}{r}$ = number of possible combinations of n objects taken r at a time
$P(E)$ = probability of the event E
$P(E \mid H)$ = conditional probability that E occurs given that H has occurred
p_{ij} = transition probability
$\hat{\theta}$ = estimator of θ
μ = statistical mean
σ^2 = statistical variance
$\frac{\partial}{\partial \theta}$ = differentiation with respect to θ

Subscripts

0 = for a GO status
1 = for a NOGO status
00 = GO given GO
10 = GO given NOGO
01 = NOGO given GO
11 = NOGO given NOGO

Superscript

(2) = pertaining to two transitions (days)

Introduction

SPACE-VEHICLE mission operations are subject to several atmospheric processes, such as thunderstorm, precipitation, cloud ceiling, peak surface wind speed, etc. These atmospheric parameters are constraints on the mission operations. An atmospheric parameter is a random variable which is assigned the values of either 0 or 1, for GO or NOGO outcomes, respectively. In repeated observations each one of these atmospheric parameters embodies a certain dependence structure. In fact, a meteorological observation is usually not independent of the preceding conditions. However, the dependence decreases as the length of the time interval between successive events increases. For example, the amount of rain in a month is influenced to a small but definite extent by the amount of rain in the preceding month, but the amount of rain in a year bears practically no relation to the amount of rain in the preceding year. In daily observations the interdependence is found to be even more marked. This is due to the fact that rain tends to persist from day to day. Thus, in general, it is the characteristic of meteorological events to stick together; high or low values tend to occur in clusters rather than asisolated incidents.

An important part of mission planning is being able to provide, ahead of time, a good assessment of the GO–NOGO status for different atmospheric parameters as well as conditional probabilities involving GO and/or NOGO outcomes. Specifically, it is of interest to effectively address certain questions pertaining to the assigned constraints for the different mission phases of the space vehicle; see Smith and Batts.[1] Examples of the issues in such questions are as follows:

1) Finding the probability that the assigned atmospheric constraints will (or will not) occur for N consecutive days, at a particular time of the day.

2) Given that an assigned constraint has occurred (or has not occurred) for n consecutive days, at a particular time of the day, finding the probability that it will continue for N additional days.

3) Estimating the probability distribution of certain sequences of GO and NOGO outcomes.

4) Estimating certain conditional probabilities involving GO and/or NOGO outcomes.

Effectively addressing and giving specific answers to these types of questions is useful in many ways, for instance, 1) determining design criteria for the space vehicle, 2) establishing flight operational rules as well as launch commit criteria, and 3) setting up effective cost assessments.

The procedure currently used to obtain atmospheric statistics for aerospace vehicle operations is based on empirical formulas and ad hoc methods. Thus, the need for a unified approach to utilize methods substantiated by well-established theory was identified. The purpose of this paper is to establish analytical models substantiated by sound theoretical foundation, and use these models to answer the types of questions raised above. The theoretical foundation in this work involves the theory of Markov chains with two states and the theory of runs; see for example Feller[2] and Wilks.[3]

Repeated observations of any given atmospheric parameter are not usually independent, because a meteorological observation depends to some extent on the preceding conditions. Therefore, the underlying dependence structure in the model is a crucial aspect in the development of our investigation. Based on the nature of the meteorological observations, it seems reasonable to utilize a dependence structure of the Markovian type. Markovian dependence has been used to model daily rainfall occurrence; for example, see Ref. 4. This model was shown in that work to give a good fit to various aspects of rainfall occurrence patterns. We should point out here that several authors used the geometric distribution, the negative binomial, or related distributions as models for meteorological activity and wet–dry cycles of rain. For example, a distribution of weather cycles by length was investigated using the geometric

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distribution; see Ref. 5. We note that these distributions do arise under the Markovian dependence structure. Of course, they may also originate under different circumstances.

The theory of Markov chains is well established in the literature. Here we use a Markov chain with two states. Consider a sequence of random variables $X_0, X_1, \ldots$, and suppose that the set of possible values of these random variables is $\{0, 1\}$. Then X_n is interpreted as the state of some system at time n, and, in accordance with this interpretation, we say that the system is in state i at time n if $X_n = i$. The sequence of random variables is said to form a Markov chain if each time the system is in state i there is some fixed probability p_{ij} that it will next be in state j. The values $p_{ij}, 0 \leq i, j \leq 1$, are called the transition probabilities of the Markov chain. The transition probabilities p_{ij} can be arranged in a square array $\mathbb{P}$, which is called a transition probability matrix, as follows:

$$\mathbb{P} = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} \tag{1}$$

Knowledge of $\mathbb{P}$ and the distribution of X_0 enables us, at least in theory, to compute all probabilities of interest. For example, suppose that if it is GO today, then it will be GO tomorrow with probability p_{00}, and if it is NOGO today, then it will be GO tomorrow with probability p_{10}. This is a two-state Markov chain having transition probability matrix given by Eq. (1). If it is GO today, the probability that it will be GO two days from now is

$$p_{00}^{(2)} = p_{00}^2 + p_{01}p_{10} \tag{2}$$

The remaining probabilities $p_{01}^{(2)}, p_{10}^{(2)}$, and $p_{11}^{(2)}$, which are the entries of $\mathbb{P}^2$, can be given similar interpretations.

The methods discussed in this article are illustrated using days as time transition periods. However, these methods are equally applicable if the transition periods are hours or a particular hour of the day. In that case hourly atmospheric data should be used.

Probability Models

Consider a sequence of random variables each of which takes either the value 0 or 1 (for GO or NOGO, respectively). We do not assume that these random variables are independent. The basic assumption about their interdependence is that, given the present value, the future and the past values are independent. Thus the value of a random variable at time $i+1$, X_{i+1}, depends only on its value at time i, X_i. This is a Markovian dependence structure, and such a probabilistic model is referred to as a Markov chain with two states $\{0, 1\}$.

A Markov probability model with two states usually possesses two parameters; see Ref. 6. These two parameters can be defined in different ways. Two versions are considered here. In model 1 we take the two parameters as the conditional probabilities

$$\begin{aligned} \theta_1 &= P(X_i = \text{NOGO} \mid X_{i-1} = \text{NOGO}) \\ &= P(X_i = 1 \mid X_{i-1} = 1), \qquad i = 2, \ldots, n \end{aligned} \tag{3}$$

and

$$\begin{aligned} \theta_0 &= P(X_i = \text{NOGO} \mid X_{i-1} = \text{GO}) \\ &= P(X_i = 1 \mid X_{i-1} = 0), \qquad i = 2, \ldots, n \end{aligned} \tag{4}$$

In model 2 we define the two parameters as follows:

$$\begin{aligned} p &= P(X_i = \text{NOGO}) \\ &= P(X_i = 1), \qquad i = 2, \ldots, n \end{aligned} \tag{5}$$

and

$$\begin{aligned} \lambda &= P(X_i = \text{NOGO} \mid X_{i-1} = \text{NOGO}) \\ &= P(X_i = 1 \mid X_{i-1} = 1), \qquad i = 2, \ldots, n \end{aligned} \tag{6}$$

The conditional probabilities above measure the degree of dependence or persistence (or lack of it) in the chain, and the probability in Eq. (5) is the usual frequency parameter. The parameter p in model 2 is easily shown to be related to the parameters θ_1, θ_0 in model 1 as follows:

$$p = \frac{\theta_0}{1 + \theta_0 - \theta_1} \tag{7}$$

Therefore we only use model 1 in our discussion. The transition probability matrix takes the form

$$\mathbb{P} = \begin{bmatrix} 1-\theta_0 & \theta_0 \\ 1-\theta_1 & \theta_1 \end{bmatrix} \tag{8}$$

Clearly, this model reduces to the case of independent Bernoulli trials if $\theta_0 = \theta_1$.

The parameters of either model can be estimated by using the appropriate relative frequency.[4] Here, we use a modified maximum-likelihood method which was used by P. Billingsley in his development of the asymptotic theory of maximum-likelihood estimators.[7] Therefore, this method is particularly useful when n is large. The full likelihood in terms of θ_0 and θ_1 can be written as

$$\begin{aligned} &P(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = P(X_1 = x_1) \\ &\quad \times P(X_2 = x_2 \mid X_1 = x_1) P(X_3 = x_3 \mid X_2 = x_2) \cdots \\ &\quad \times P(X_n = x_n \mid X_{n-1} = x_{n-1} = p^{x_1}(1-P)^{1-x_1} \\ &\quad \times \prod_{i=2}^{n} \theta_1^{x_{i-1}x_i}(1-\theta_1)^{x_{i-1}(1-x_i)} \theta_0^{(1-x_{i-1})x_i}(1-\theta_0)^{(1-x_{i-1})(1-x_i)} \end{aligned} \tag{9}$$

The factor $p^{x_1}(1-p)^{1-x_1}$ represents the contribution due to the first state visited by the process. The likelihood is modified by neglecting that factor. Now, the natural logarithm of the likelihood, denoted by L, for the realizations $x_1, x_2, \ldots, x_n$ can be expressed as

$$L = x_1 \ln p + (1 - x_1) \ln(1-p) + L'$$

where

$$L' = n_{00} \ln(1-\theta_0) + n_{01} \ln \theta_0 + n_{10} \ln(1-\theta_1) + n_{11} \ln \theta_1 \tag{10}$$

The $n_{ij}, i, j = 0, 1$, are the usual transition counts given by the number of indices m for which $x_m = i$ and $x_{m+1} = j, m = 1, \ldots, n-1$, so that $n_{00}+n_{01}+n_{10}+n_{11} = n-1$. Therefore, the required estimators of θ_0 and θ_1 are obtained by maximizing L' given by Eq. (10):

$$\frac{\partial L'}{\partial \theta_0} = \frac{n_{00}}{1-\theta_0}(-1) + \frac{n_{01}}{\theta_0} = 0$$

yields

$$\hat{\theta}_0 = \frac{n_{01}}{n_{00} + n_{01}} \tag{11}$$

and

$$\frac{\partial L'}{\partial \theta_1} = \frac{n_{10}}{1-\theta_1}(-1) + \frac{n_{11}}{\theta_1} = 0$$

yields

$$\hat{\theta}_1 = \frac{n_{11}}{n_{10} + n_{11}} \tag{12}$$

For model 2, we can get the estimators of p using Eq. (7), to obtain

$$\hat{p} = \frac{\hat{\theta}_0}{1 + \hat{\theta}_0 - \hat{\theta}_1} \tag{13}$$

where $\hat{\theta}_0$ and $\hat{\theta}_1$ are given in Eq. (11) and Eq. (12), and it simplifies to

$$\hat{p} = \frac{n_{10}(n_{10} + n_{11})}{n_{00}n_{10} + 2n_{01}n_{10} + n_{01}n_{11}} \tag{14}$$

Distribution of GO–NOGO States

The models discussed above can easily be used to give the probability distributions of GO and/or NOGO states.

Distribution of the Number of GO states

Let X be the number of successive GO states until a NOGO state occurs. Then

$$P(X = n) = (1 - \theta_0)^{n-1}\theta_0, \qquad n = 1, 2, \ldots \tag{15}$$

Distribution of the Number of NOGO States

Similarly, if Y is the number of successive NOGO states until a GO state occurs, then

$$P(X = m) = \theta_1^{m-1}(1 - \theta_1), \qquad m = 1, 2, \ldots \tag{16}$$

Distribution of Recurrence Time

A succession of NOGO states of length k, $k \geq 0$, means a sequence of k NOGO states preceded and followed by GO states. Therefore, a NOGO succession of length k is equivalent to a recurrence time of $k + 1$ for GO states.

Recurrence Time of NOGO States

Table 1 illustrates how the probabilities of different recurrence of NOGO states are computed. From it, the mean and variance of the recurrence time T_1 of NOGO states are given by

$$\mu_1 = \frac{1 - (\theta_1 - \theta_0)}{\theta_0} \tag{17}$$

and

$$\sigma_1^2 = \frac{1 - \theta_1}{\theta_0^2}(1 + \theta_1 - \theta_0) \tag{18}$$

Recurrence Time of GO States

Similarly we show the recurrence time of GO states in Table 2. The mean and variance of the recurrence time T_0 of GO states can be computed from Table 2 and are given by

$$\mu_0 = \frac{1 - (\theta_1 - \theta_0)}{1 - \theta_0} \tag{19}$$

and

$$\sigma_0^2 = \frac{\theta_0}{(1 - \theta_1)^2}(1 + \theta_1 - \theta_0) \tag{20}$$

Asymptotic Distribution of NOGO–GO States

Since NOGO and GO states are represented by 1 and 0, respectively, it follows that the total number of NOGO states in a sequence of n trials $\{X_1, X_2, \ldots, X_n\}$ is $S = \sum_{k=1}^{n} X_k$. It is shown in Feller[2] that S can be approximated by a normal distribution provided that the number of observations n is large enough. In this case the approximate mean and variance are n/μ_1 and $n\sigma_1^2/\mu_1^3$, respectively, where μ_1 and σ_1^2 are given by Eqs. (17) and (18). Corresponding formulas for the recurrence time of GO states can similarly be obtained by using μ_0 and σ_0 instead of μ_1 and σ_1, respectively. However, these results neither tell how rapid the distributions approach normality nor reflect the exact distributions for small n. The exact distribution will be discussed in the Analysis of Runs section.

Table 1 Recurrence time of NOGO

k	Recurrence time of NOGO	Representation	Probability
0	1	11	θ_1
1	2	101	$(1 - \theta_1)\theta_0$
2	3	1001	$(1 - \theta_1)(1 - \theta_0)\theta_0$
3	4	10001	$(1 - \theta_1)(1 - \theta_0)^2\theta_0$
$\vdots$	$\vdots$	$\vdots$	
k	$k+1$	$100\cdots01$	$(1 - \theta_1)(1 - \theta_0)^{k-1}\theta_0$

Table 2 Recurrence time of GO

k	Recurrence time of GO	Representation	Probability
0	1	100	$1 - \theta_1$
1	2	010	$\theta_0(1 - \theta_1)$
2	3	0110	$\theta_0\theta_1(1 - \theta_1)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	$k+1$	$011\cdots10$	$\theta_0\theta_1^2(1 - \theta_1)$

Distribution of Successive GO Followed by Successive NOGO States

Let X be a random variable representing the number of successive GO states, and let Y be a random variable representing the number of successive NOGO states. Then X and Y are independent random variables. Define $Z = X + Y$. Then Z represents a number of successive GO states followed by a number of successive NOGO states, and

$$\begin{aligned} P\{Z = n\} &= \sum_{k=1}^{n-1} P(X = k)P(Y = n - k) \\ &= \sum_{k=1}^{n-1} \theta_1^{k-1}(1 - \theta_1)(1 - \theta_0)^{n-k-1}\theta_0 \\ &= \theta_0(1 - \theta_1)\sum_{k=1}^{n-1} \theta_1^{k-1}(1 - \theta_0)^{n-k-1} \\ &= \theta_0(1 - \theta_1)\frac{(1 - \theta_0)^{n-1} - \theta_1^{n-1}}{(1 - \theta_0) - \theta_1} \end{aligned} \tag{21}$$

We note that the distribution of Eq. (21) is symmetric in θ_0 and $1 - \theta_1$; this is expected, because $Z = X + Y = Y + X$.

Analysis of Runs

We consider here sequences of NOGO and GO states. In our treatment here the sequences of NOGO and GO states are not assigned equal probabilities. For a complete discussion of analysis of runs of two kinds of elements where all sequences are assigned equal probability we refer the reader to Wilks.[3]

First we set our notation. Suppose that the total number of GO states is n_0 and that of NOGO states is n_1, with $n_0 + n_1 = n$. The class of all these sequences is that of all $\binom{n}{n_0}$ permutations of n_0 GO states and n_1 NOGO states. Each sequence consists of runs of NOGO and GO states. The length of a run is the number of states in it. Let r_{0k} denote the number of runs of GO states of length k, and let r_{1j} denote the number of runs of NOGO states of length j. For example, the sequence 00011001001101 is such that $n_0 = 8$, $n_1 = 6$, $r_{01} = 1$, $r_{02} = 2$, $r_{03} = 1$, $r_{11} = 2$, $r_{12} = 2$, and all other r's are zero. Let $r_0 = \sum r_{0k}$ and $r_1 = \sum r_{1j}$ be the total numbers of runs of GO and NOGO states, respectively. The total number of ways of having r_{0k} runs of GO states of lengths $k = 1, 2, \ldots, n_0$ and r_{1j} runs of NOGO states of lengths $j = 1, 2, \ldots, n_1$ is

$$\frac{r_0!r_1!\delta(r_0, r_1)}{r_{01}!r_{02}!\cdots r_{0n_0}!r_{11}!r_{12}!\cdots r_{1n_1}!}$$

where $\delta(r_0, r_1)$ is the number of ways of arranging r_0 indistinguishable objects of one kind and r_1 indistinguishable objects of a second kind so that no two objects of the same kind appear together; then $\delta(r_0, r_1)$ takes the value 0, 1, or 2, according as $|r_0 - r_1| > 1, = 1$, or $= 0$.

To find the probability of a GO–NOGO sequence, consider a sequence with r_{0k} runs of GO states of lengths $k = 1, 2, \ldots, n_0$ and r_{1j} runs of NOGO states of lengths $j = 1, 2, \ldots, n_1$. In order to obtain the probability $P(E)$, of getting such a sequence, we start by conditioning on the state of the first trial. That is, letting H denote the event that the first trial results in a NOGO, we then have

$$P(E) = pP(E \mid H) + (1 - p)P(E \mid H^c) \tag{22}$$

where $p = \mathrm{P(NOGO)} = \theta_0/(1 + \theta_0 - \theta_1)$, and

$$\begin{aligned} P(E \mid H) &= \theta_1^{n_1 - r_1}(1 - \theta_0)^{n_0 - r_0}(1 - \theta_1)^{r_1 + \delta - 2}\theta_0^{r_0 - \delta + 1} \\ &= \theta_1^{n_1 - r_1}(1 - \theta_0)^{n_0 + 1}\left(\frac{1 - \theta_1}{\theta_1}\right)^{r_1 - 2} \\ &\times \left(\frac{\theta_0}{1 - \theta_1}\right)^{r_0 + 1}\left(\frac{1 - \theta_1}{\theta_0}\right)^{\delta} \end{aligned} \tag{23}$$

The factor $\theta_1^{n_1-r_1}$ accounts for all the consecutive NOGO states, $(1-\theta_0)^{n_0-r_0}$ accounts for all the consecutive GO states, and $(1-\theta_1)^{r_1+\delta-2}$ accounts for the number of changes from a NOGO to a GO (or from a1 to a0) state. We note that if $r_0 = r_1$, then $\delta = 2$ and there are r_1 changes from a NOGO to a GO state. On the other hand, if $r_1 > r_0$, then $\delta = 1$ and there are $r_1 - 1$ such changes. The case $r_1 < r_0$ cannot occur, by the definition of runs, since we begin with a run of NOGO states. Finally, the factor $\theta_0^{r_0-\delta+1}$ accounts for the number of changes for a GO to a NOGO (or from a0 to a1) state. We observe here that if $r_0 = r_1$, then $\delta = 2$ and there are $r_0 - 1$ changes from a GO to a NOGO state, while there are r_0 changes from a GO to a NOGO state if $r_1 > r_0$.

Similarly, given that the initial state is a GO state, then

$$P(E \mid H^c) = (1-\theta_0)^{n_0-r_0}\theta_1^{n_1-r_1}\theta_0^{r_0+\delta-2}(1-\theta_0)^{r_1-\delta+1}$$

$$= (1-\theta_0)^{n_0-2}\theta_1^{n_1+1}\left(\frac{\theta_0}{1-\theta_0}\right)^{r_0-2}$$

$$\times\left(\frac{1-\theta_1}{\theta_1}\right)^{r_1+1}\left(\frac{\theta_0}{1-\theta_1}\right)^{\delta} \tag{24}$$

Substituting Eqs. (23) and (24) in Eq. (22), the probability P(E) of a given sequence with r_{0k} runs of GO states of lengths $k = 1, 2, \ldots, n_0$ and r_{1j} runs of NOGO states of lengths $j = 1, 2, \ldots, n_1$ is

$$P(E) = p\theta_1^{n_1-2}(1-\theta_0)^{n_0+1}\left(\frac{1-\theta_1}{\theta_1}\right)^{r_1-2}\left(\frac{\theta_0}{1-\theta_0}\right)^{r_0+1}$$

$$\times\left(\frac{1-\theta_1}{\theta_0}\right)^{\delta} + (1-p)(1-\theta_0)^{n_0-2}\theta_1^{n_1+1}\left(\frac{\theta_0}{1-\theta_0}\right)^{r_0-2}$$

$$\times\left(\frac{1-\theta_1}{\theta_1}\right)^{r_1+1}\left(\frac{\theta_0}{1-\theta_1}\right)^{\delta} \tag{25}$$

where $p = \theta_0/(1+\theta_0-\theta_1)$

Therefore, the probability of the sequence of NOGO and GO states as described above is given by

$$\frac{r_0!r_1!\delta(r_0, r_1)}{r_{01}!\cdots r_{0n_0}!r_{11}!\cdots r_{1n_1}!}P(E) \tag{26}$$

The probability formula obtained in Eq. (26) can be thought of as a joint p.d.f. of runs of NOGO and GO states. Hence, other marginal distributions can be obtained by summing Eq. (26) over the appropriate indices. For example, the joint p.d.f. of r_0 GO and r_1 NOGO can be shown to be equal to

$$P(r_0, r_1) = \binom{n_0-1}{r_0-1}\binom{n_1-1}{r_1-1}\delta(r_0, r_1)P(E) \tag{27}$$

Conditional Probabilities

The conditional probabilities involved in the types of questions mentioned in the introduction can be computed easily using the models discussed above. Suppose we know that a GO state has occurred, at a particular time of the day. What is the probability that the GO state will continue for N additional days? Using the Markov property along with Eqs. (3) and (4), we see that

$$P(\text{GO} = N \mid \text{GO}) = (1-\theta_0)^N \tag{28}$$

Similarly,

$$P(\text{GO} = N \mid \text{NOGO}) = (1-\theta_1)(1-\theta_0)^{N-1} \tag{29}$$

Likewise,

$$P(\text{NOGO} = N \mid \text{GO}) = \theta_0\theta_1^{N-1} \tag{30}$$

and

$$P(\text{NOGO} = N \mid \text{NOGO}) = \theta_1^N \tag{31}$$

Results and Discussion

We now illustrate the methods discussed above, using 33 years of data for thunderstorm and nonthunderstorm days for the summer months of June, July, and August at Kennedy Space Center (KSC); see Tables 3 and 4. A similar analysis can be performed for any other atmospheric parameter of concern.

Parameter Estimation

To estimate the parameters θ_1, θ_0 we use Eqs. (11) and (12), where n_{11} is the frequency of having two consecutive NOGO days (i.e., two back-to-back thunderstorm days), n_{00} is the frequency of having two consecutive GO days (i.e., two back-to-back nonthunderstorm days), n_{01} is the frequency of having a GO day followed by a NOGO day, and n_{10} is the frequency of having a NOGO day followed by a GO day. From Table 3, $n_{11} = 859$ and $n_{01} = 1024$. From Table 4, $n_{00} = 1328$ and $n_{10} = 1024$. Therefore, $\hat{\theta}_1 = 859/(1024+859) = 0.4562$, and $\hat{\theta}_0 = 1024/(1328+1024) = 0.4354$. The significance of the estimates $\hat{\theta}_1, \hat{\theta}_0$ is that we have actually summarized the 33 years of data in just two numbers.

We observe here that in estimating the conditional probabilities θ_1 and θ_0 we used both Tables 3 and 4, i.e., the data for thunderstorm

Table 3 Thunderstorm days at KSC, June, July, August, 1957–1989

Run length	Frequency
1	206
2	118
3	56
4	41
5	28
6	14
7	18
8	6
9	7
10	3
11	0
12	2
13	1
14	1
15	1
16	2

Table 4 Nonthunderstorm days at KSC, June, July, August, 1957–1989

Run length	Frequency
1	180
2	85
3	67
4	48
5	41
6	26
7	15
8	14
9	14
10	5
11	5
12	4
13	5
14	2
15	2
16	3
17	2
18	0
19	1
20	0
21	0
22	0
23	0
24	1

days as well as the data for nonthunderstorm days. This is because the thunderstorm days and the nonthunderstorm days are "elusively correlated." An important feature of the methods in this article is that the correlation between GO and NOGO days is taken into consideration. This was not done in the method suggested in Ref. 1.

For the parameter p, we use Eq. (13) or Eq. (14) to obtain

$$\hat{p} = \frac{\hat{\theta}_0}{1 + \hat{\theta}_0 - \hat{\theta}_1} = \frac{0.4354}{1 + 0.4354 - 0.4562} \approx 44.5\%$$

where $\hat{p}$ represents the probability of NOGO due to thunderstorm activities.

Conditional Probabilities

1) The probability of n successive GO states until a NOGO state occurs [using Eq. (15)] is

$$0.4354(1 - 0.4354)^{n-1}, \qquad n = 1, 2, \ldots \tag{32}$$

2) The probability of m successive NOGO states until a GO state occurs [Eq. (16)] is

$$(1 - 0.4562)(0.4562)^{m-1} = 0.5438(0.4562)^{m-1}, \quad m = 1, 2, \ldots \tag{33}$$

3) By Eqs. (28–31) given that a GO day has occurred, the probability that the GO state will continue for N additional days is

$$P(\text{GO} = N \mid \text{GO}) = (1 - 0.4354)^N \tag{34}$$

4) Similarly,

$$P(\text{GO} = N \mid \text{NOGO}) = 0.5438(1 - 0.4354)^{N-1} \tag{35}$$

$$P(\text{NOGO} = N \mid \text{GO}) = 0.4354(0.4562)^{N-1} \tag{36}$$

and

$$P(\text{NOGO} = N \mid \text{NOGO}) = (0.4562)^N \tag{37}$$

where $N = 1, 2, 3, \ldots$.

Fit of the Model

Table 5 shows the number of thunderstorm days as well as the relative frequencies of occurrence of thunderstorms in June, July, and August during 1957–1989 compared with $\hat{p}$, the probability of NOGO due to thunderstorm activities during this time period, computed from the model. In fact a χ^2 test, comparing the annual observed frequencies with the expected frequencies for 33 years, shows the theoretical model provides a good fit at significance level of 0.005.

Table 5 Relative frequency of thunderstorm days at KSC in June, July, August, 1957–1989

	June	July	August
No. of thunder-storm days	410	496	441
Relative frequency	0.414	0.485	0.431
Estimated $\hat{p}$	0.445	0.445	0.445

Concluding Remarks

Probabilistic models incorporating random dependence structure have been presented. These models are substantiated by the theory of Markov chains and the theory of runs. Using the models the probabilities of different configurations of GO and/or NOGO can be estimated. Thus the methods and techniques developed could be used in mission planning by providing, ahead of time, a good assessment of GO–NOGO decisions related to weather conditions in the different phases of a mission.

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